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THE THEORY OF AN AUTO-RESONANT FIELD EMISSION CATHODE RELATIVISTIC ELECTRON ACCELERATOR FOR HIGH EFFICIENCY MICROWAVE TO DIRECT CURRENT POWER CONVERSION

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A novel method of microwave power conversion to direct current is discussed that relies on a modification of well known resonant linear relativistic electron accelerator techniques. In particular, an analysis will be presented that shows how, by establishing a "slow" electromagnetic field (in particular, a traveling TE wave) in a waveguide, electrons liberated from an array of field emission cathodes (FEC's), are resonantly (with respect to the phase of the electromagnetic wave) accelerated to several times their rest energy, thus establishing an electric current over a large potential difference. Such an approach is not limited to the relatively low frequencies that characterize the operation of rectennas (i.e., 2.45 GHz) and can, with appropriate waveguide and slow wave structure design, be employed in the 300-600 GHz range where much smaller transmitting and receiving antennas are needed.

1. INTRODUCTION

The advent of future space and planetary missions brings to the fore the usual energy transfer problems, e.g., launching payloads form earth or from other planetary surfaces, LEO to GEO orbital transfer and, especially with such proposed missions as the Mars mission, the transfer of energy, in particular, in the form of electrical power, to the surface of planets to support the various organized exploration activities (for example, powering the "Mars Rover"). The need to distribute energy from a minimal number of centralized sources (e.g., from a nuclear reactor to an ion engine-driven LEO to GEO space vehicle, or from an orbiting reactor to several widely separated planetary exploration sites and/or an electrically driven rover) in an efficient manner suggests the reconsideration of microwave power transmission and its related conversion from and to useful electrical power. In the past, microwave power transmission concepts were fraught with one major problem, the source of which is directly attributable to the particular microwave to DC conversion device used, i.e., the rectenna. The problem encountered is that unrealistically large transmitting and receiving antennas need to be used for efficient transfer of the microwave radiation; the source of this obstacle is that the rectennas employed (and the only efficient ones still available) work in the 2.45 GHz (i.e., ~ 10 cm wavelength) band.

The purpose of this paper is to demonstrate the theoretical feasibility of the basis of a novel microwave power conversion scheme that is not bound to such a low frequency of operation. Here, the resonant relativistic acceleration of electrons, obtained from an FEC array in a waveguide by action of the electric field of the EM wave field, accelerated by the electric and magnetic fields of the same EM wave field allows one to obtain potential differences on the order of several million volts. Of course, the true test of any conversion scheme is the overall conversion efficiency that prevails. However, this will by highly dependent on the design of the waveguide and the field emission cathode within it. This aspect of the conversion scheme will not be addressed here but incentive will certainly be provided for such a study by the results given here.

2. DESCRIPTION OF THE OVERALL POWER CONVERSION SCHEME

The overall power conversion scheme envisioned here is as follows: a waveguide is fitted with a slow wave structure and a field emission cathode. The FEC can in the form of a helix subtending an angular range of 2π radians with a pitch that is commensurate with that of the traveling, circularly polarized wave the electric field of which induces the electron field emission. In the presence of an appropriately specified, externally applied magnetic field (it will be shown in Section 3 why this external field is necessary) the electrons are then accelerated in the transverse plane of the waveguide by the electric field of the wave while, at the same time, the transverse momentum is converted into longitudinal momentum by the attendant magnetic field of the wave. Resonance (or actually, anti-resonance) of the accelerating electrons with the oscillating electric field of the wave is constantly maintained by the external magnetic field. A point is reached where all of the transverse motion of the electrons is converted into longitudinal motion; at this point the electrons are collected, thus establishing an electric current across an induced potential difference in a circuit joining the collector and waveguide wall. The concept is depicted in Figure 1.

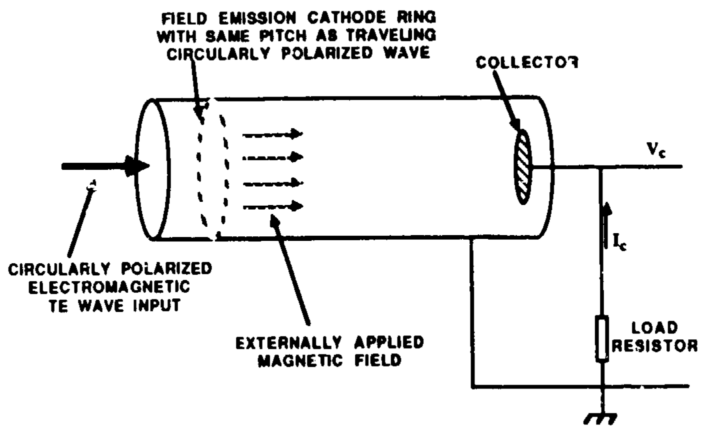

FIGURE 1.
SLOW WAVE FIELD-EMISSION ELECTRON ACCELERATOR FOR HIGH EFFICIENCY MICROWAVE TO DIRECT-CURRENT POWER CONVERSION

In what is to follow in Section 3 below, the theoretical basis of the foundation of the proposed conversion method will be developed under rather palatable assumptions. Even though the acceleration of the field emission electrons relies on a properly designed slow wave structure and waveguide, these latter elements will not be addressed here since they already represent a well developed area of methodology. However, such considerations are necessary for a complete assessment of conversion efficiencies; only with a complete accounting of the various losses in the slow wave / waveguide system can one calculate the available electric field intensities at the FEC's, the beam loading power, etc. Thus, the analysis below will provide a calculation of the potential difference V_C realized by this method of power conversion. The attendant beam current I_C, easily found from the well known Fowler-Nordheim equation, and the and the related converted power $P_C = V_C I_C$ will be functions of the particular slow wave structure and waveguide design and will be considered in a future study. For the same reasons, analyses concerning the maximum transverse and longitudinal dimensions will not be included in the discussion below, although it is not too difficult to obtain these results.

3. THEORETICAL DEVELOPMENT OF THE ACCELERATION TECHNIQUE

Consider a circularly polarized TE wave traveling along the axial direction of a circular waveguide in the presence of an externally applied magnetic field that is also directed along the axis. (The fact that this must be a slow TE wave will be established below.) Let there exist at the input to the waveguide an appropriately arranged field emission cathode array from which electrons are continuously liberated by the action of the electric field of the wave. Since the outcome of the acceleration technique is the increase of electron energies to several times their rest energies, a relativistic description of motion is in order. Neglecting 1) the presence of the necessary radial variation of the external magnetic field and 2) electron interactions and the action of "beam currents" on the traveling wave, the equation of motion of an emitted electron in such a situation is

$$\frac{d\mathbf{p}}{dt} = e\left[\left(1 - \frac{\beta_z}{\beta_{ph}}\right)\mathbf{E}\right] + \frac{e}{c\beta_{ph}}(\mathbf{v} \cdot \mathbf{E})\hat{z} + \frac{e}{c}\mathbf{v} \times \mathbf{B}_e \tag{3.1}$$

where $\mathbf{p}$ is the momentum and e is the elementary charge of the electron, c is the velocity of light, $\mathbf{v}$ is the velocity of the electron, $\beta_z = v_z/c$ for the component of $\mathbf{v}$ in the longitudinal z direction ($\hat{z}$ is the unit vector in this direction), $\beta_{ph} = v_{ph}/c$ where v_{ph} is the phase velocity of the wave, $\mathbf{E}$ is the electric field attendant with the TE wave, and $\mathbf{B}_e$ is the externally applied magnetic field the spatial form of which, at this point, remains to be determined. The first term on the right side of Eq.(3.1) describes the transverse force caused by the combined electric field of the wave and the Lorentz force of the magnetic field of the wave, the second term describes the longitudinal Lorentz force due to the magnetic field of the wave, and the third term is the total force that results from the external magnetic field. It is through this "externally induced" force that one can design and control the acceleration process. Also, from considerations of total energy of the electron, one has the relation

$$m_0c^2\frac{d\gamma}{dt} = e\mathbf{v} \cdot \mathbf{E} \tag{3.2}$$

where $\gamma = 1/\sqrt{(1-v/c)}$. Adopting plane polar coordinates in the transverse plane of the electron motion, Eqs.(3.1) and (3.2) give

$$m_0\frac{d}{dt}\left(\gamma v_\perp\right) = e\left(1 - \frac{\beta_z}{\beta_{ph}}\right)\mathbf{E} + \frac{e}{c}B_d\left(\rho\theta'\rho - \rho'\theta\right) \tag{3.3}$$

$$m_0\frac{d}{dt}\left(\gamma z'\right) = \frac{e}{c\beta_{ph}}\mathbf{E}\cdot \mathbf{v} \tag{3.4}$$

$$m_0c^2\frac{d\gamma}{dt} = e\mathbf{E}\cdot \mathbf{v} \tag{3.5}$$

where the primes denote differentiation with respect to time, m_0 is the rest mass of the electron and $v_\perp = \rho'\rho + \rho\theta'\theta$ is the transverse velocity of the electron. The last two equations yield a prevailing constant of motion, I, viz,

$$\gamma\left(1 - \beta_z\beta_{ph}\right) = I \tag{3.6a}$$

This constant can easily be specified by the initial conditions that exist for an electron of initial total energy γ_0 and initial velocity β_{z0} in the longitudinal z direction; hence, one has

$$\gamma\left(1 - \beta_z\beta_{ph}\right) = \gamma_0\left(1 - \beta_{z0}\beta_{ph}\right) \tag{3.6b}$$

At this point, it is convenient to represent the circularly polarized traveling TE wave as well as the transverse velocity as

$$\mathbf{E} = E\left(\cos\theta_w\rho + \sin\theta_w\theta\right), \quad \theta_w = \omega\left(t - \frac{z}{c\beta_{ph}}\right) + \phi_w \tag{3.7}$$

$$v_\perp = v_\perp\left(\cos\theta_p\rho + \sin\theta_p\theta\right), \quad \theta_p = \omega\left(t - \frac{z}{c\beta_{ph}}\right) + \phi_p \tag{3.8}$$

where ω is the angular frequency of the electromagnetic radiation, ϕ_w is the phase of the wave and ϕ_p is that of the particle. Substituting Eqs.(3.7) and (3.8) into Eq.(3.3), expanding the derivative on the left side and simplifying yields a relation that governs the relative phases of the wave and particle motions, i.e.,

$$\frac{d\left(\phi_p - \phi_w\right)}{dt} = \omega\left(1 - \frac{\beta_z}{\beta_{ph}}\right) - \frac{\omega_B}{c\gamma} - \frac{1}{\gamma v_\perp}\left(\frac{d\gamma v_\perp}{dt}\right)\tan\left(\phi_p - \phi_w\right) \tag{3.9}$$

where ω_B is the cyclotron frequency of the electron in the B-field given by

$$\omega_B = \frac{eB_e}{m_0}$$

The relationship of Eq.(3.9) demonstrates the fact that if the condition

$$\omega\left(1 - \frac{\beta_z}{\beta_{ph}}\right) = \frac{\omega_B}{c\gamma} \tag{3.10}$$

is enforced throughout the motion, and if $\tan(\phi_p-\phi_w) = 0$ initially, the electron and TE wave will remain in resonance (or anti-resonance). If this is the case, then for an electron introduced into the crossed fields within the waveguide by field emission from an appropriately arranged cathode induced by the attendant electric field. $\phi_p-\phi_w = \pi$, anti-resonance is thus achieved, and as will be shown below, the electron will be accelerated in the longitudinal direction.

Assuming that the condition of Eq.(3.10) prevails throughout the motion of the electron and, hence, the electron remains in anti-resonance with the electric field of the wave, and taking the scalar product of Eq.(3.3) with $\gamma v_\perp$ and simplifying gives

$$m_0\frac{d\gamma v_\perp}{dt} = e\left(1 - \frac{\beta_z}{\beta_{ph}}\right)E\cos\left(\phi_p-\phi_w\right) = -\,e\left(1 - \frac{\beta_z}{\beta_{ph}}\right)E \tag{3.11}$$

showing that the electron is accelerated toward the axis of the waveguide so long as $\beta_z<\beta_{ph}$; when the condition $\beta_z>\beta_{ph}$ prevails, the electron reverses its direction of acceleration. But as found from Eq.(3.5),

$$\frac{d\gamma}{dt} = \frac{e}{m_0c^2}Ev_\perp\cos\left(\phi_p-\phi_w\right) = -\frac{e}{m_0c^2}Ev_\perp \tag{3.12}$$

Hence, as indicated by Eqs.(3.12) and (3.13), the electron energy will not only increase when $\beta_z<\beta_{ph}$ but continue to increase when $\beta_z>\beta_{ph}$ so long as $v_\perp < 0$, i.e, the electron continues to move toward the axis of the waveguide. With $v_\perp = 0$, the acceleration process stops and the the direction of motion is totally along the axis of the waveguide making electron collection straightforward. This situation can only be realized when one deals with a slow TE wave where $\beta_{ph} < c$.

Expressions for the evolution of the energy and the longitudinal velocity can be obtained from Eqs.(3.3) and (3.5) but are unwieldy and, in this case where electron interactions and the attendant beam currents are neglected, unnecessary to use. One can easily obtain such relationships from the integral of motion, Eq.(3.6b), and the well known relativistic identity

$$\gamma^2\left(1-\beta_\perp^2-\beta_z^2\right)=1 \tag{3.13}$$

Soiving these equations for β_z yields

$$\beta_z=\frac{\beta_{ph}\pm\gamma_0\left(1-\beta_{z0}\beta_{ph}\right)\sqrt{\left[\gamma_0^2\left(1-\beta_{z0}\beta_{ph}\right)^2+\beta_{ph}^2\right]\left(1-\beta_\perp^2\right)-1}}{\gamma_0^2\left(1-\beta_{z0}\beta_{ph}\right)^2+\beta_{ph}^2} \tag{3.14}$$

The choice of which root to use in this solution is dictated by the initial conditions; at the point of field emission of the electron into the crossed fields, $\beta_\perp << 1$ and $\beta_{z0} \approx 0$. Therefore, one has $\gamma_0 \approx 1$ and Eq.(3.14) becomes $\beta_z = (\beta_{ph} \pm \beta_{ph})/(1+\beta_{ph}^2)$ thus indicating that the "−" sign be used for values of $\beta_\perp$ up to the point where the radical vanishes. This consideration also indicates that if the end of the acceleration process is defined where $\beta_\perp = 0$, the maximum longitudinal velocity attained by the electron is

$$\beta_{z,\,max}=\frac{2\beta_{ph}}{1+\beta_{ph}^2} \tag{3.15}$$

The vanishing of the expression under the radical in Eq.(3.14) defines the maximum value of the transverse velocity $\beta_{\perp max}$ that is attained by the electron; equating this expression to zero and solving for $\beta_\perp$ yields

$$\beta_{\perp max}=\sqrt{\frac{\gamma_0^2\left(1-\beta_{z0}\beta_{ph}\right)^2+\beta_{ph}^2-1}{\gamma_0^2\left(1-\beta_{z0}\beta_{ph}\right)^2+\beta_{ph}^2}} \tag{3.16}$$

After this point where the two roots coincide, the root defined by the "+" sign prevails. The transverse velocity will begin to decline from $\beta_{\perp max}$ but the longitudinal velocity will continue to increase. The value of the declining $\beta_\perp$ at which $\beta_z = \beta_{ph}$ is found by applying this constraint to Eq.(3.14) and solving for $\beta_\perp$ which gives

$$\beta_\perp = \sqrt{\frac{\left(I^2+\beta_{ph}^2-1\right)\left[I^2-\beta_{ph}^2\left(I^2+\beta_{ph}^2-1\right)\right]}{I^2\left(I^2+\beta_{ph}^2\right)}} \qquad (3.16)$$

where $I = \gamma_0(1-\beta_{z0}\beta_{ph})$. The fact that the event $\beta_z = \beta_{ph}$ occurs after $\beta_{\perp max}$ is realized is also borne out in the equations of motion, viz, Eqs.(3.11) and (3.12). Expanding the differentiation on the left side of Eq.(3.11) and using Eq.(3.12) yields

$$\gamma\frac{d\beta_\perp}{dt} = -\frac{eE}{m_0c}\left[1-\frac{\beta_z}{\beta_{ph}}-\beta_\perp^2\right]$$

Since $d\beta_\perp/dt = 0$ at $\beta_\perp = \beta_{\perp max}$, one has

$$\beta_z = \beta_{ph}\left(\frac{I^2+\beta_{ph}^2-1}{I^2+\beta_{ph}^2}\right) < \beta_{ph}$$

This provides the key behavior of the electron in the acceleration process: The transverse velocity of the electron increases due to the action of the electric field of the traveling wave. Of course, some of the attendant transverse momentum gained by the electron is converted to longitudinal momentum by the action of the magnetic field of the wave but that in the transverse direction predominates over that in the longitudinal direction. At the transverse velocity $\beta_{\perp max}$, the situation reverses where the longitudinal momentum increases faster than that of the transverse. The longitudinal velocity continues to increase at the expense of the transverse until $\beta_\perp = 0$.

Using Eq.(3.15) with Eq.(3.6b) yields an expression for the maximum energy gained by the electron, i.e.,

$$\gamma_{max} = \frac{1+\beta_{ph}^2}{1-\beta_{ph}^2} \qquad (3.17)$$

Finally, it must be remembered that the entire analysis presented above presupposed the fact that the resonance condition of Eq.(3.10) prevails throughout the acceleration process. With the values of β_z and γ evolving during acceleration, it is apparent that the externally applied magnetic field must also change accordingly. Eliminating the factor γ from Eq.(3.10) using Eq.(3.6) and solving for B_e yields

$$B_e = \frac{m_0 c\omega}{e}\left(\frac{I}{1 - {}^{\wedge}\beta_{ph}}\right)\left(\frac{\beta_{ph} - \beta_z}{\beta_{ph}}\right) \tag{3.18}$$

in the general case and in the case of field emission considered above, I=1. In this specific case with $\beta_z \approx 0$ initially, $B_e = B_{e0} = m_0 c\omega/e$. When $\beta_z = \beta_{ph}$, $B_e = 0$. Finally, at the other extreme where $\beta_z = \beta_{z,\,max}$ as given by Eq.(3.15), one has $B_e = -B_{e0}$. Thus, the external magnetic field must initially have an intensity of B_{e0} and monotonically decrease to zero as the point where $\beta_z = \beta_{ph}$ is reached, and then change direction and monotonically increase to a value of $-B_{e0}$.

As an example of the parameters and specifications encountered in the acceleration of field emission electrons within a waveguide by a traveling TE wave, consider the case where β_{ph}=0.9 at an operating frequency of 100 GHz ($\omega = 6.28 \times 10^{10}$ rad./sec). From Eqs.(3.15) and (3.17) one has $\beta_{z,\,max} = 0.9945$ and $\gamma_{max} = 9.53$. Thus, within the constraints of the theory presented above, an electron can be accelerated to 9.53 times its rest energy or 4.86 Mev. The maximum value of the externally applied longitudinal magnetic field needed to achieve the anti-resonance of the traveling electric field and accelerating electron os found to be B_{e0} = 100.7 kG.

Electrons accelerated in such a manner will induce a potential difference V_C between the FEC array and the collector. Since V_C must be such that the quantity eV_C is the total kinetic energy of the electrons, i.e.,

$$V_C = \frac{m_0 c^2 (\gamma_{max} - 1)}{e} \tag{3.19}$$

one has from this relation and Eq.(3.17)

$$V_C = m_0 c^2 \left(\frac{2\beta_{ph}^2}{1 - \beta_{ph}^2}\right) \tag{3.20}$$

From the values calculated above, one finds that $V_C = 4.35 \times 10^6$ volts can be realized.

Thus, the possibility of accelerating field emission electrons in a waveguide by the same electromagnetic field from which field emission was attained has been demonstrated. The key to the operation of such an acceleration scenario is the fact that momentum gained by the electron from the transverse electric field of the wave is converted into longitudinal momentum by the action of the attendant magnetic field of the wave. The process continues until all longitudinal momentum has been depleted. The ability for the accelerating electron to remain in synchronization with the traveling wave during this process is provided by an externally applied magnetic field that satisfies a specific intensity profile along the axis of the waveguide.

In the analysis above, consideration was not given to the transverse and longitudinal dimensions over which the acceleration process must take place. This requires a much more detailed exposition than that given here but will certainly be addressed in future publications on this subject. The purpose here was just to demonstrate the theoretical possibility of such a technique for power conversion purposes.

4. CONCLUSIONS NAD SUGGESTIONS FOR FURTHER WORK

It has been shown that under the rather non-restrictive assumptions that one neglect the ever-present transverse variation of the constant external B-field (at least in the paraxial region) and that the motion of the electrons are independent of one another and do not act back on the traveling wave, one can obtain linear acceleration of field emitted electrons tc several times their rest energies. There is, however, another element of the process that has not been addressed; that is the stability of the accelerated electron motion with respect to the variations of the dynamical parameters of the field emission electrons. In the above treatment, it was only assumed that $\beta_{\perp 0} << 1$ and $\beta_{z0} \approx 0$. A rigorous analysis would require the full solution of Eqs.(3.3)-(3.5) (or Eqs.(3.6), (3.11), and (3.12) in light of the resonance condition Eq.(3.10)) with perturbations introduced into the initial conditions. One can then study the behavior of the electrons as they make the transition to stable motion. This, as well as the incorporation of the possibility of electron interactions into the theory, should be the next level of approximation that is considered so a thorough comparison can be made with the foregoing.